Learning from experts to aid the automation of proof search

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Motivation

- Most formal methods give rise to POs.
- $5 - 20\%$ of these will (still) require interactive proofs.
- In industrial scale developments a large amount of user expertise is required to manually discharge POs.
- Proof automation is a bottleneck in such developments.
- Many of these proofs will be of **similar form**.
- Models **change** during development
  - existing proofs may change as a result of model changes.

Research Hypothesis

- The user manually proves one exemplar proof.
- **Consequently:** once the hurdle of this exemplar proof is passed then proof of POs in the same "family" can be discharged by "manipulating" the exemplar proof.
- **Research hypothesis:** By learning from the exemplar proof we can automatically discharge the POs of "similar form".
  - This also implies more robustness to model changes.

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- **Research hypothesis:** By learning from the exemplar proof we can automatically discharge the POs of "similar form".
  - This also implies more robustness to model changes.
- Targets **real industrial needs** as exemplified yesterday:
  - **Mathieu Clabaut (Systerel):** Proof engineering
    - design proof for **management** & **reuse**
  - **Andreas Roth (SAP)**
    - guided tool usage: "*... verification tools may fail because the problem is not typical...*"
    - the need for robust tools.

Model-based formal development methods

- Model-based formal development methods are remarkably similar[1].
- For example, a reification/refinement step requires
  - VDM retrieve function
  - (classical) B linking invariant
  - Event-B gluing invariant
  - Morgan's coupling invariant.
- All of them give rise to (similar type of) POs.
- Our proposal targets increased automation for most model-based formal development methods.

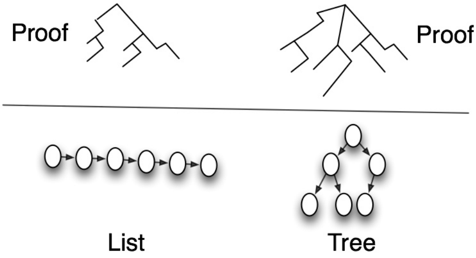
[1] see Formal Aspects of Computing Vol. 20, No. 1, 2008.

Approaches to automation

- Model checking/SMT solvers
  - automatic "push-button" techniques
  - limited expressiveness.
- Theorem provers
  - search space too large for complete search
  - most contain heuristics to guide search
  - user-interaction still required for $5 - 20\%$ of POs.
- We aim to complement these techniques
  - by targeting those $5 - 20\%$ of POs
  - achieved by exploring new information from an exemplar proof
  - and reuse this to automatically discharge "similar" POs.

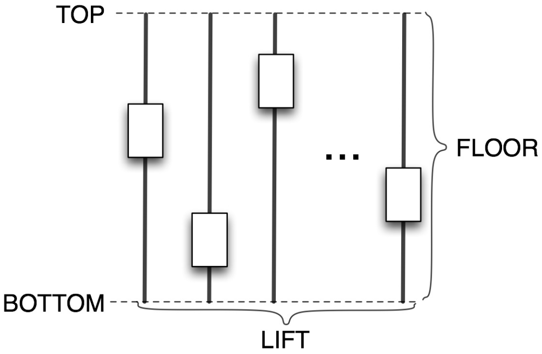
"Re-use" of proof

- "Similar form" does not mean same proof
  - different lemmas required
  - different instantiations
  - different data structures
  - ...



A lift example in Event-B: specification

$$\begin{array}{ll} axiom1: & \forall f \cdot (BOTTOM \leq f \wedge f \leq TOP) \Leftrightarrow (f \in FLOOR) \\ inv1: & lift \in LIFT \longrightarrow FLOOR \end{array}$$



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$$axiom1: \quad \forall f \cdot (BOTTOM \leq f \wedge f \leq TOP) \Leftrightarrow (f \in FLOOR)$$
$$inv1: \quad lift \in LIFT \longrightarrow FLOOR$$

EVENT *move_up* $\widehat{=}$
  ANY l
  WHERE
    $grd1: \ l \in LIFT$
    $grd2: \ lift(l) < TOP$
  THEN
    $act1: \ lift(l) := lift(l) + 1$

EVENT *move_down* $\widehat{=}$
  ANY l
  WHERE
    $grd1: \ l \in LIFT$
    $grd2: \ lift(l) > BOTTOM$
  THEN
    $act1: \ lift(l) := lift(l) - 1$

A lift example in Event-B: POs

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$$\begin{array}{l} lift \in LIFT \longrightarrow FLOOR \\ l \in LIFT \\ lift(l) < TOP \\ \vdash \\ lift \mathbin{\lhd\!\!-} \{l \mapsto lift(l) + 1\} \\ \quad \in LIFT \longrightarrow FLOOR \end{array}$$

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Proof outline:

- lemma $lift(l)+1 \in FLOOR$
- inst $[lift(l)+1/f]$ in *axiom1*

$$\begin{array}{l} lift \in LIFT \longrightarrow FLOOR \\ l \in LIFT \\ lift(l) > BOTTOM \\ \vdash \\ lift \lhd\!\!- \{l \mapsto lift(l) - 1\} \\ \quad \in LIFT \longrightarrow FLOOR \end{array}$$

Proof outline:

- lemma $lift(l)-1 \in FLOOR$
- inst $[lift(l)-1/f]$ in *axiom1*

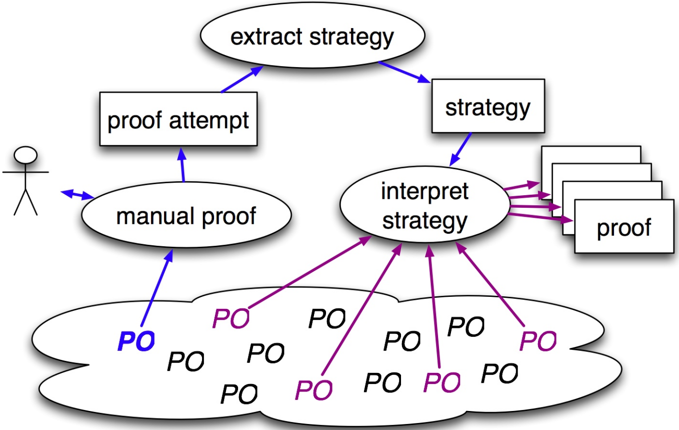
"Re-use" of proof: properties

- Not striving for automatic detection of similar "claims"
  - differs from analogy-based reasoning.
- Not expecting a fixed set of "patterns"
  - the user must still provide an exemplar proof.
- Low-level LCF tactics are too brittle
  - behaves differently for similar POs
  - not sufficiently robust to cope with changes
  - something higher-level is required...
  - ...as in proof planning...
  - however, we need to go beyond that...

Towards a strategy language

- We will explore additional information from the exemplar proof.
- Classification is a key challenge
  - as in exemplar-based reasoning
  - however, user-supplied exemplar proof required.
- Classification of proofs
  - data structures
  - shape of PO
  - information from PO Generator
    - e.g. refinement proof, invariance proof, ...
- A combination of a high-level proof strategy
- ... with a "vocabulary" of terms
  - instantiated using data structure information.

Illustration of strategy language



Evidence for strategy language: rippling

- **Rippling** provides evidence for such a high-level strategy language.
- Rippling is a **proof plan**
  - AI technique for mechanising formal reasoning based upon high level proof patterns
- Great **flexibility**, e.g. productive use of failure
- Wide range of applications
  - software/hardware verification, synthesis, correction of faulty specifications, ...
  - promotes **reuse** across domains
- Provides a starting point for the strategy language.

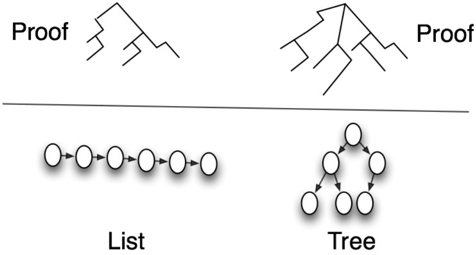
Strategy language properties (1)

- "Standard" proof plans and patches
  - exemplar proof will typically exhibit a new plan or a patch to an existing plan
  - we also aim to capture "dead ends" in proof attempts and expert recovery.
- Choices of unusual induction rules and variables
  - choice of an alternative induction rule/variable as a patch.
- Intermediate lemmas
  - construct and prove key intermediate lemmas.
- Generalisation
  - generalise the original PO as a patch.
- Insert case-analysis.

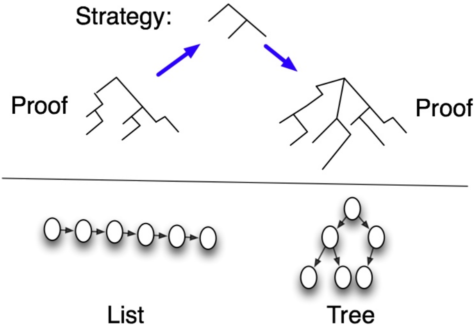
Strategy language properties (2)

- "Similar proofs" will deviate more than patterns currently captured in proof planning & rippling
  - higher level of abstraction.
- We expect to use additional kinds of abstraction
  - e.g. explore various dimensions of information
    - data structure, form, POG, ...
- The strategy language must be sufficiently **expressive**, **robust** and **general-purpose**
  - it must be able to deal with unanticipated proof plans and patches.
- We will use generic taxonomies
  - type of induction rule
  - type of generalisation
  - ...

"Re-use" of proof revisited



"Re-use" of proof revisited



A sequence of lemmas

- A strategy may describe the sequence of intermediate lemmas required to be proved:

  prove lemma L_1
  then prove lemma L_2
  ⋮
  then prove lemma L_n
  then GOAL follows from $L_1 \cdots L_n$
- Similar to the "ACL2 approach"
  - experiences from the Piton project[2] confirms to our hypothesis
- At a "reasonable" level, discharge using
  - e.g. a (correct) decision procedure
  - with e.g. a time-out feature.
- Requires a notion of a **gap**
  - an unproved sub-goal/lemma.

[2]see J S. Moore. *Piton: A Mechanically Verified Assembly-Level Language*.

Our plan

- We plan to analyse a lot of
  - POs,
  - their expert-provided proofs attempts,
  - and their "families".
- Based on analysis develop a strategy language.
- Provide tool support
  - to extract a strategy out of an exemplar proof
  - to interpret strategies to discharge "similar POs".
- There are a lot of challenges
  - classification of POs
  - creating a strategy language
  - deriving strategies from proofs
  - interpreting strategies for the same "family".

Conclusion

- Significant proof automation can be achieved by learning from one proof to discharge "similar" POs.
- The use of such proof strategy approach is robust to model changes.
- We believe this is applicable to most model-based top-down formal development methods
  - ... and possibly to bottom-up approaches as well.
- There is a strong industrial & academic need for such tool
  - e.g. Systerel, SAP, ACL2 (Piton), ...