Models of “why”

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Contents

AI4FM project

Report to date

- archaeology on proofs
- new proofs (with introspection) for "process"
- agreed to look at "How to say why"
  - record "intent" rather than details
  - more transferable than steps
  - hope to record "process"
- examples (next slides)
- theme taken up:
  - nice (internal) notes on this from Alan, Leo, . . .
  - these have certainly influenced what follows
- Leo reads!
  - Lenat's AM, . . . , HR, Argunet
  - supports/informs what follows

examples

- typically, want higher level than “case split”
  - “set up induction”
  - operator order mismatch
  - try for a normal form
- choice of lemmas — some pro-actively generated
  - “shape” of lemmas — copied twixt theories
- J’s “extract toy problem” as cutting down the search space
- Alan has a whole set from equation solving
- . . .
- overall aim: cut search space

My "Σ approach"

- new name — been doing it for decades!
- sketch a state (Σ from habit)
  - quickly pinpoints questions
  - basis for discussion
- *mural* started life as a VDM model
  - . . . as have many more studies
- what follows is pretty much a first attempt
  - (although Leo has endured some even rougher versions)
  - it will evolve!
- what I'm searching for is a *framework* to test ideas
- at this stage:
  - question is how to record higher level proof views
  - come later to how to learn them

State (i)

Assume state contains various $Theory$s

$$\Sigma :: Id \xrightarrow{m} Theory$$

Theories are organised hierarchically in two senses:

$$\begin{aligned} Theory :: & \cdots \\ & types\colon Id\text{-set} \\ & operators\colon Operator \xrightarrow{m} OpDefn \\ & results\colon Id \xrightarrow{m} Conjecture \\ & \cdots \end{aligned}$$

Operators/functions merged into $OpDefn$

State (ii): Conjectures

$$\begin{aligned} Conjecture :: \; & hypotheses\text{: } Expr^* \\ & goal\text{: } Expr \\ & justification\text{: } Id \xrightarrow{m} \textsc{Axiom} \mid Attempt \mid \cdots \\ & \cdots \end{aligned}$$

Plus notions like $complete$

Will also store negative results: $DisProof$s

State(iii)

. . . extending $Conjecture$

$Conjecture$ :: $\cdots$
$shape$: $MetaType$
$uses$: $Clue$-set

record "intent" (and learn)

$Clue$ :: $intent$: Why
$evidence$: $Test$-set

$Why = \text{COMMUTEOPERANDS}, \text{NORMALFORMREDUCTION}, \cdots,$
$\text{DISTRIBUTEOPERATORS}, \text{MAPTOANOTHERDATATYPE}, \cdots$

$Test$ :: $predicate$: $Expr^* \times Expr \times Conjecture$-set $\rightarrow \mathbb{B}$
$weight$: $\mathbb{Z}$

State(iv)

. . . extending $Theory$

$$
\begin{aligned}
Theory &:: \cdots \\
&\quad strategies: Id \xrightarrow{m} Strategy \\
\\
Strategy &:: split: Conjecture \to Conjecture\text{-set} \\
&\quad combine: Conjecture\text{-set} \to Attempt \\
&\quad \cdots \\
&\quad needs: Why\text{-set} \\
&\quad weight: \mathbb{N}
\end{aligned}
$$

Some possible scenarios

generated POs come in as $Conjecture$s — incomplete!
models themselves will often give rise to new $Theory$
for any new $OpDefn$ (pro-actively):

- . . .
- sibling $Theory$s could suggest lemmas by analogy
  - (thanks Leo!)
  - cf. $MetaType$
- decide whether to try automatic proof immediately
  - another role for learning
- even if proof fails, keep statement of the putative lemma
  - . . . in fact, if counter-example found, store $DisProof$
  - (thanks Aaron)
- using parallelism/concurrency

More scenarios

for any incomplete $Conjecture$:

- check if something "matches" . . .
- $analyse$ to get a $Diffn$ set twixt $from/to$
- $\{$OPERATORORDER, DIFFERENTOPERATORS, $\exists$NEEDED$\} \subseteq Diffn$
- some $Diffn$s prompt searching for a $Conjecture$
  e.g. OPERATORORDER points to COMMUTEOPERANDS or DISTRIBUTEOPERATORS
- other $Diffns$ might match $Strategy$s
  e.g. $\exists$NEEDED might need FINDWITNESS
- use $evidence$ to decide which avenue to try first

What we are doing *now*

- *not* aiming at general maths proofs
  - precisely: FM POs (from "posit & prove" style)
- "how to say why" + "models of why"
  - can crystalise debate around Σ
  - . . . refutable!
- remember — at this stage of AI4FM:
  - think about how to record high-level proof strategies
  - the issue of how to learn them comes next
  - $weight$s above are but a small nod to learning!
- proof tracing (Andrius)
  - Rodin tools
  - Isabelle
- inclusion of (parallel) "disproving"