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Productive use of failure in formal methods

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- Proposed work
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Background – Formal methods

- Formal methods: mathematical approach
- How to apply FMs:
  - bottom-up: for existing system; debugging
  - top-down: towards codes; posit and prove; cost-effective
- Some techniques can be used in top-down e.g. Z, B, Event-B, VDM and so on …

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Background – Proof obligations

- Justifications generate proofs obligations (POs)
- POs are putative lemmas require proofs in theorem prover, e.g. Z/Eves for Z; Rodin for B/Event B
- 5 - 10% POs require human interaction
  - e.g. the Paris Metro line 14; 2,250 / 27,800 POs
- How to discharge them
  - by the modelling strategy
  - by the proof strategy
- These POs are expensive to discharge
- Proof automation becomes a bottleneck in industry.

Background – An example of POs

Telephone exchange in Z - states

Exchange

$Free, Unavailable, Callers: \mathbb{P}\ Subs$ -- Sets of Subscribe numbers

$call: Subs \pfun SubRec$ -- Calls' num maps to (call's status, callee's num)

$connected: Subs \rightarrowtail Subs$ -- Call num maps to callee num

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$\langle Free, Unavailable, \mathrm{dom}\ call \cup \mathrm{ran}\ connected \rangle$ partition $Subs$

$Callers = \mathrm{dom}\ ((call \fatsemi st) \rhd Connected)$ -- a set of status e.g. ringing, speech

$connected = Callers \lhd (call \fatsemi num)$

Background - An example of POs

Telephone exchange in Z – an operation

LiftFree

$\Delta Exchange$

$s? : Subs$

$s? \in Free$

$Free' = Free \setminus \{s?\}$

$Unavailable' = Unavailable$

$call' = call \cup \{(s? \mapsto (seize, \langle\rangle))\}$

Background - An example of POs

Telephone exchange in Z - a PO

$$Callers = \mathrm{dom}\,((call \fatsemi st) \rhd Connected)$$

$$s? \in Free \quad call' = call \cup \{(s? \mapsto (seize, \langle\rangle))\}$$

To prove:

$$s? \in Free \Rightarrow Callers = \mathrm{dom}\,(call' \fatsemi st) \rhd Connected$$

Which is:

$$s? \in Free \Rightarrow Callers = \mathrm{dom}\,((call \cup \{(s? \mapsto (seize, \langle\rangle))\}) \fatsemi st) \rhd Connected$$

Proposed work - Outline

- Our hypothesis is
  - *It is possible to classify FM PO into families.*
  - *In each family there exist patterns of proof strategies.*
  - *The strategy can be used to guide the proof search of the blocked proofs from the same family.*
- Key words: *family, failure.*

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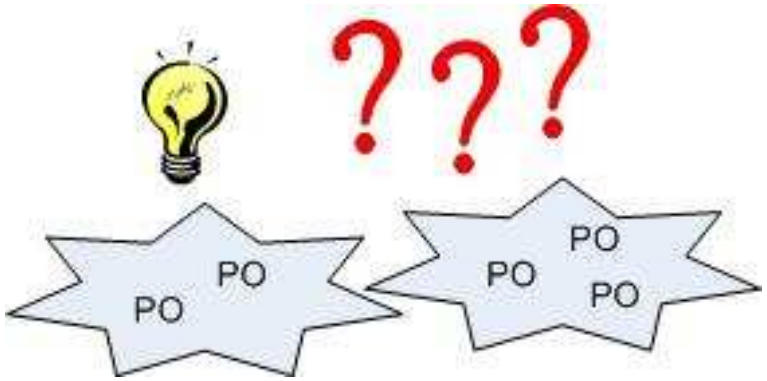
Proposed work – key word family

- Definition for *family* from Merriam-Webster

  a group of things related by common characteristics

Can POs in FMs be classified into families?



Proposed work - Families of POs

- Definition for *family* from Merriam-Webster

  a group of things related by common characteristics

- Nature of POs in FMs:
  - the number of POs is huge.

    e.g. the Paris Metro line 14; 2,250 / 27,800 POs
  - POs tends to exhibit a “similarity”

    e.g. an industrial case-study using Event-B, 100 / 300 POs, can be handled by 5 strategies.

Proposed work – An example for *family*

- Proofs for PO1

PO1:

$$s? \in Free \Rightarrow Callers = \mathrm{dom}\ ((call \cup \{(s? \mapsto (seize, \langle\rangle))\}) \fatsemi st) \rhd Connected$$

From: $s? \in Free \Rightarrow s? \notin dom\ (call)$

Hence: $Callers = dom\ ((call \fatsemi st) \cup (\{(s? \mapsto (seize, \langle\rangle))\} \fatsemi st)) \rhd Connected$

Hence:

$$Callers = dom\ (((call \fatsemi st) \rhd Connected) \cup ((\{(s? \mapsto (seize, \langle\rangle))\} \fatsemi st) \rhd Connected)$$

Hence: $Callers = dom\ (((call \fatsemi st) \rhd Connected) \cup \varnothing)$

Equals invariant: $Callers = \mathrm{dom}\ ((call \fatsemi st) \rhd Connected)$

Proposed work – An example for *family*

- Proofs for PO2

Inv: $connected = Callers \lhd (call \fatsemi num)$

PO2: $s? \in Free \Rightarrow connected = Callers \lhd (call' \fatsemi num)$

Which is: $connected = Callers \lhd ((call \cup \{(s? \mapsto (seize, \langle\rangle))\}) \fatsemi num)$

Equals: $connected = Callers \lhd ((call \fatsemi num) \cup (\{(s? \mapsto (seize, \langle\rangle))\} \fatsemi num))$

Equals: $connected = (Callers \lhd (call \fatsemi num)) \cup (Callers \lhd (\{(s? \mapsto (seize, \langle\rangle))\} \fatsemi num))$

From: $(Callers \lhd (\{(s? \mapsto (seize, \langle\rangle))\} \fatsemi num)) \Rightarrow Callers \lhd \{s? \mapsto \langle\rangle\} \Rightarrow \varnothing$

Done !

Proposed work – An example for *family*

- Similarities of proofs between PO1 and PO2

PO1: $s? \in Free \Rightarrow Callers = \mathrm{dom}\ (call' \fatsemi st) \rhd Connected$

PO2: $s? \in Free \Rightarrow connected = Callers \lhd (call' \fatsemi num)$

These two POs can be discharged in one strategy, which is to "push" changes to the outer level until it can be showed that the changes are empty.

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Proposed work - key word *failure*

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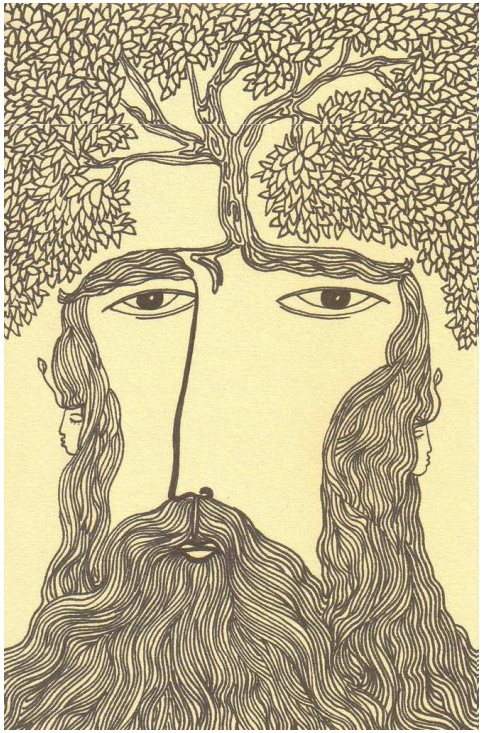
Proposed work - key word *failure*

Some encouraging sayings:

- Nothing fails like success because we don't learn from it. We learn only from failure. ~Kenneth Boudling
- There is much to be said for failure. It is more interesting than success. ~Max Beerbohm
- Failure is only the opportunity to begin again more intelligently. ~Henry Ford



Well…but

they are not for mathematics.

We are dealing with proofs !!!

Proposed work – Use of failures in proofs

- Rippling: is a rewriting technique for the automation of mathematical reasoning which works when the goal is embedded in one of the givens.
  - not exhaustive rewriting, skeleton preserving and measure decreasing
  - rewriting in either direction
  - guarantee termination
- Proof critics : productive use of failure in rippling
  - from strong expectation of rippling, it can suggest a missing lemma, generalisation or a different induction rule.

Proposed work – proposed use of failure

- Gain value of failures by
  - analysing why they fail, how they fail.
  - extract meta-level strategies from patches to guide search of proof in similar case (family)
- Moreover, if a strategy fails in the families, critics can be developed to handle it as an "exception"

Proposed work – Approach

- Empirical process:

  Collect POs and patches from existing industrial examples, e.g. Tokeneer

- Iterative development:
  - Strategies between families are supposed to be relatively independent
  - Implement strategies as methods (most likely in IsaPlanner)
  - Develop patches to failed strategies within families of POs as critics

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Summary

- Undischarged POs are a bottleneck of development.
- We proposed to extract proof strategies for families of POs by analysing failures and patches.
- POs in FMs are suitable to be classified into families.
- Learning from failures can help us improve power to discharge POs automatically
- Our development is empirical and iterative.

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Thank you !!!

Q&A

- This work is supported by EPSRC grant EP/H024204/1

- AI4FM: using AI to aid automation of proof search in Formal Methods.
- For more details see http://www.ai4fm.org.